Unit: **2-D and 3-D Measurement**
Lesson: **3-D Measurement Review**

1. Is there meaningful choice in learning for students?
2. Are student differences considered in planning (e.g. readiness/learning style)?
3. How is the learning made accessible/engaging for all students (e.g. open tasks with multiple entry points)?

❑ Graph Calc
❑ ti-navigator
❑ Alg Tiles
❑ 1 cm Link Cubes
❑ 2 cm Link Cubes
❑ Geo-Solids
❑ Venn Diagram
❑ x-y grid
❑ pattern blocks
❑ chart papr/mkrs
❑ fraction circles
❑ fraction strips
❑ geoboard
❑ angle arm
❑ rulers

Math Learning Goals

1. Solve measurement problems with 3-D shapes
2. Make connections between equivalent measurement representations (i.e. between algebra and physical or pictorial representations)
3. Solve and evaluate equations in a measurement context with and/or without technology

MFM1P Overall Expectations
❑ Solving Problems Involving Proportional Reasoning
❑ Simplifying Expressions and Solving Equations
❑ Using Data Management to Investigate Relationships
❑ Determining Characteristics of Linear Relations
❑ Connecting and Solving Problems With Linear Representations
❑ Optimal Values of Measurements of Rectangles
❑ Solving Problems With Perimeter, Area, and Volume
❑ Investigating and Applying Geometric Relationships

Minds On...

1. Character Education introduction. At the start of the period, or as students enter class, they respond to an "Agree-Disagree" continuum on a character quotation of the day using the interactive whiteboard. The quote for this lesson was: "They may forget what you said, but they will never forget how you made them feel" by Carl W. Buechner. Students have choice in participating or not participating and if they participate they record the degree to which they agree or disagree with the statement on the interactive whiteboard. A brief discussion of the quotation follows with students. This introduction sets the stage for DI, students have meaningful choice, all opinions are valued and every student can contribute to our learning.

2. The formal math lesson begins with the teacher identifying the specific learning goals of the day, written on the whiteboard, connecting to big ideas of the unit which are posted on chart paper. There is explicit modeling of how to connect representations using a "visualizing" strategy. The teacher performs a "think-aloud" to model the meta-cognitive process used while visualizing to make explicit connections between representations in measurement, specifically, connecting area formulas of composite shapes to their pictorial representations. Note that the minds-on activity to come for students is 3-D whereas the "think-aloud" is for 2-D. This is purposeful to have them transfer the visualizing strategy from 2-D to 3-D. See Page 2 of the Interactive whiteboard File.

3. "I have…who has" minds-on activity where the students apply a visualizing strategy to connect algebraic and 3-D representations. This is a cooperative and differentiated activity using manipulatives. Half of the students have cards with algebraic representations and the other half have equivalent 3-D model representations. Students have to find a partner with an equivalent representation by making connections. The representations varied in complexity and were assigned based on student readiness. To summarize, partners justify, using appropriate vocabulary and mathematical reasoning, why their representations are equivalent.

❑ Narrative – Tell/read a story
❑ Narrative – Pose questions, problem
❑ Logical/Quant – Analyze graph or table
❑ Logical/Quant – graphic organizer concepts
❑ Foundational – Explain theory, big questions
❑ Aesthetic – Picture, video
❑ Experiential – experiment, game, manipulatives
❑ Placemat
❑ Anticipation Guide
❑ Tiered Prior Knowledge Questions

❑ Choise ❑ Readiness ❑ Interest ❑ Learning Style (MI, VAK) ❑ Intelligence Preference (triarchic)

Action!

Learning Centres

Students pre-select two of three learning centres the day before this lesson based on the "People-Data-Things" differentiation structure. Learning activities earlier in this unit are organized with the People, Data and Things framework to allow students to develop a working knowledge of the differentiation structure and provide opportunities to build an understanding of their own strengths and interests related to this differentiation structure. The three learning centres are:

1. People (supportive of students with interest/strength in interpersonal intelligence/auditory learning style) – Peer coaching cooperative activity with students consolidating their understanding of solving versus evaluating equations in a measurement context.
2. Things (supportive of students with interest/strength in bodily intelligence/kinesthetic learning style) – Problem solving in measurement using manipulatives.
3. Data (supportive of students with interest/strength in spatial or logical/mathematical intelligence/visual learning style) – Technology is used to make connections between representations.

Materials and handouts are set out at three locations. Please see the attached handouts for the specific details of each centre. The teacher identifies locations of the centres and reminds students of key learning attributes for each station. For example, at the "People" learning centre, review attributes of an effective "math coach" e.g. supportive, identifies mistakes but does not correct mistakes when coaching (i.e. questions their partner to lead them towards correcting their work). Students complete two of three learning centres during the class.

Assessment for learning occurs as the teacher circulates between the stations. Learning centres afford an opportunity to assess the Learning Skills especially "Working Independently" and "Teamwork".

- ❑ Tiering – Par'll Tasks
- ❑ Cooperative Learning
- ❑ Learning Centres
- ❑ Choice Boards
- ❑ Investigation
- ❑ Teaching Through Problem Solving
- ❑ Direct Instruction

Consolidate

1. To assess achievement of math goals for the day, an open question comparing representations is used. *"How are the representations below the same, different?"*

   a. $128 = \pi\,(7)^2\,(h)$ b. $V = \frac{1}{3}\pi\,(7)^2\,(12)$

This question is sufficiently open ended to allow students to compare these representations based on: cylinder in (a) versus cone in (b), solving in (a) versus evaluating in (b), determining the volume in (b) then compare volumes, identify that π is part of both equations and extend that to noting area of a circle within each equation, radius is 7 in both representations, height is unknown in (a) whereas volume is unknown in (b), they could actually solve for the height in (a) and then compare heights. Students could explore with the graphing calculator, draw pictures etc.

2. To assess effectiveness of the DI strategy, exit cards are used. Students respond and post their responses to the question *"Which center **(People, Things, Data)** today helped you learn the most about math? Why?"* The lesson is closed out by discussing exit card responses in the context of connecting learning preferences to academic success.

- ❑ Cubing
- ❑ Tiering – Par'll tasks
- ❑ Tiering – Open question
- ❑ Exit Cards
- ❑ Compare and Contrast – Venn
- ❑ RAFT
- ❑ Frayer Model (concept map)
- ❑ Tic-Tac-Toe

Represent	*Connecting*	*Selecting Tools and Strategies*	*Reasoning and Proving*	Reflecting
• mathematize a situation using concrete materials,pictures, diagrams, graphs, tables, numbers, words or algebra	• see how new concepts and skills build on old ones • apply mathematics when making connections between concepts and between math and real-life	• consider the question before I choose my computational strategy or tool • justify a strategy or use of a mathematical tool	• hypothesize and make conjectures • decide how to test my hypothesis • test my conjecture • infer, justify, and conclude	• think about reasonableness • consider the implications of data collected • self-monitor my progress

BLM 3.2 Minds-On Black line Masters "I have...Who has?"

Grade 9 MFM1P

Photo of 3-D Figures used for the Minds-On Activity

Cards for "I Have...Who Has?"

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) - \frac{1}{2}blh$$

$$V = \frac{1}{2}(\pi r^2 h) + lwh$$

Title of DI Math – 2D & 3D Measurement Lesson Plan

Grade 9 Math MFM 1P1
BLM 3.1 Understanding Me! Data, People and Things

Name: _____ List A = _____ B= _____ C = _____

Directions: Read the statements under each of the three areas and check next to the statement if you would like doing the activity described. Total up each list.

List A	List B	List C
❑ Work with calculators and computers to solve math problems. ❑ Read graphs or tables (statistics) about things that I am interested in like sports standings or lists of top movies ❑ Sort things and put them in their proper place ❑ Perform math problems accurately like figuring out the price of an item in a store. ❑ Measure things by reading scales or rulers like measuring wood for cutting or measuring cloth ❑ Design and draw plans for things like buildings, machines, or electrical systems. ❑ Look for evidence to solve problems like solving a crime. ❑ Investigate and explore the nature of the world through science and technology. ❑ Remember numbers or memorize information ❑ Compare prices in a storage or catalog and make purchases.	❑ Make people comfortable and feel safe like helping children cross the street or caring for the elderly. ❑ Perform in a skit or a play. ❑ Confront people who break rules like correcting people at school. ❑ Teach people like showing a child how to tie their shoes ❑ Help other people make decisions and solve problems by talking it out with them. ❑ Help other people when they are emotionally upset like helping a person when they have a break-up with a boyfriend/girlfriend ❑ Treat or care for sick people ❑ Help handicapped persons do such things as learn to walk, read, or talk. ❑ Give directions to people like a tour guide. ❑ Convince people to accept your opinions	❑ Mix things together to make something like bread or cement. ❑ Put things together to make things like furniture or gardens ❑ Work with hand tools like a saw, hammer, or screwdriver to make or repair things. ❑ Keep things in good working order by checking or repairing things like cars, or computers or televisions. ❑ Treat or care for animals. ❑ Do skilled art work like draw cartoons, design clothes, or paint pictures. ❑ Operate a camera or play an instrument. ❑ Invent a new machine or design a house. ❑ Use scientific equipment to run laboratory tests. ❑ Unload supplies at home or at a job.
Total Score in this column:_____	Total Score in this column:_____	Total Score in this column:_____

Source: **Http://ccdf.ca/ccdf2/cms/documents/dzozo_e.pdf**

BLM 3.3 Station 1: *People* and Practice

A _____
B _____

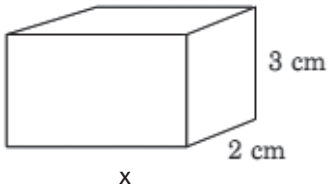
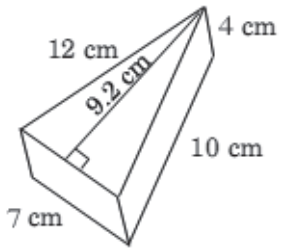
Solving and Evaluating Measurement Formulas – Peer Coaching

If the "Unknown "is already isolated after *substitution* then you are *"Evaluating"* the equation.
If the "Unknown "is **not** isolated after *substitution* then you are *"Solving"* the equation.

You will work on peer coaching in this activity.

On this first page, however, you need to identify the **error** in each solutions presented below *with your partner.* There is a mistake in each question. With your partner, identify the mistake and make the corrections for the rest of the solution. The error could be (1) selecting the wrong formula, (2) substituting in the wrong values or (3) a calculation error.

Circle the error and make corrections for each solution.

	1. **V = 39 cm³** 	Initialed by coach as correct	2. 	Initialed by coach as correct
Select formula	$V = lwh$		$V = \frac{1}{2}lwh$	
Substitute "Known" Values	$39 = (x)(2)(3)$		$V = \frac{1}{2}(4)(7)(12)$	
Identify as 1."Solving"; or 2. "Evaluating"	Solving		Eval.	
The answer – the value of the unknown	$x = 234$ cm ∴ Length = 234 cm		$V = 172$ cm³	

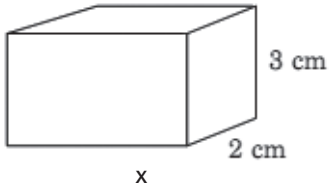
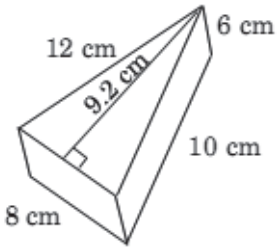
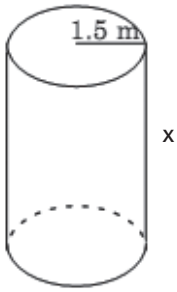
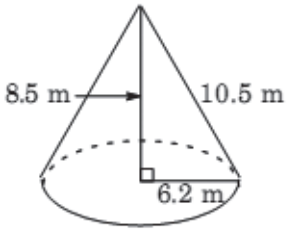
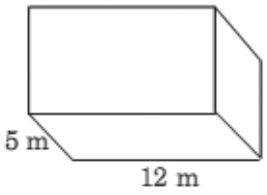
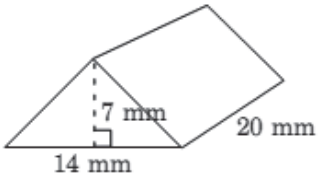
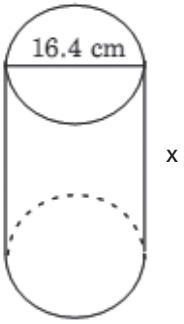
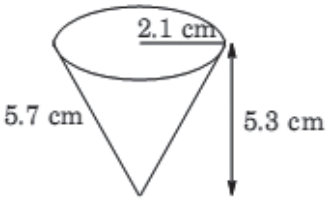
Solving and Evaluating Measurement Formulas – Peer Coaching

A _____
B _____

In each round, **"Person A"** selects a shape from **"Row A"** in the *diagrams* below then determines either the volume or the missing dimension. Person B coaches A and checks that A is correct in each step. B and A then switch roles to finish Round 1. Then move on to Rounds 2 and 3.

Diagrams

** On some diagrams, you do not need all of the dimensions given to determine the volume.
Person A: Select 3 shapes from Row A.
Person B: Select 3 shapes from Row B.

A.1 **V = 39 cm³** 	A.2 	A.3 **V = 35 cm²** 	A.4 
B.1 **V = 580 cm³** 	B.2 	B.3 **V = 4000 cm³** 	B.4 

If the "Unknown" is already isolated after *substitution* then you are *"Evaluating"* the equation or formula
If the "Unknown" is not isolated after *substitution* then you are *"Solving"* the equation or formula

Round 1	**"A" Writes with "B" Coaching**	Initialed by coach as correct	**"B" Writes with "A" Coaching**	Initialed by coach as correct
Select shape and appropriate formula				
Substitute "Known" Values				
Identify as 1."Solving"; or 2. "Evaluating"				
The answer – the value of the unknown				

Round's 2 and 3 – Peer Coaching

If the "Unknown "is already isolated after *substitution* then you are *"Evaluating"* the equation.
If the "Unknown "is not isolated after *substitution* then you are *"Solving"* the equation.

Round 2	**"A" Writes with "B" Coaching**	Initialed by coach as correct	**"B" Writes with "A" Coaching**	Initialed by coach as correct
Select shape and appropriate formula				
Substitute "Known" Values				
Identify as 1."Solving"; or 2. "Evaluating"				
The answer – the value of the unknown				

Round 3	**"A" Writes with "B" Coaching**	Initialed by coach as correct	**"B" Writes with "A" Coaching**	Initialed by coach as correct
Select shape and appropriate formula				
Substitute "Known" Values				
Identify as 1."Solving"; or 2. "Evaluating"				
The answer – the value of the unknown				

Station 2: *Data* and the Graphing Calculator

Using the Graphing Calculator to Solve Measurement Problems

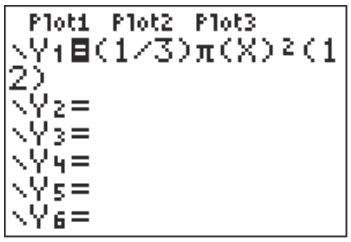
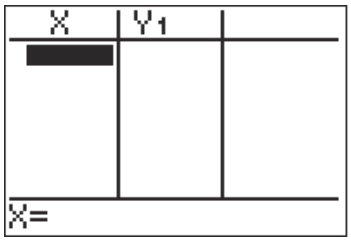
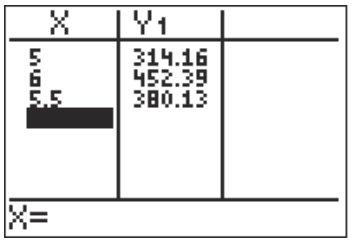
Labeled Diagram

Determine the formula and shape represented by the following equation:

$$316 = 1/3\ \pi\ x^2\ (12)$$

a. What does the 1/3 tell you? What types of volume formulae have 1/3 in them?
b. What does the "π" tell you about the type of volume formula?
c. Make a conclusion about what shape this is. Draw the shape and label the dimensions.
d. Determine the missing dimension by using the graphing calculator:

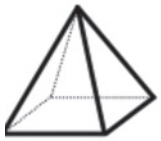
This can be solved with the "Table Feature" on the graphing calculator.

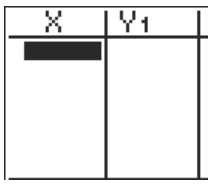
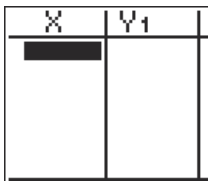
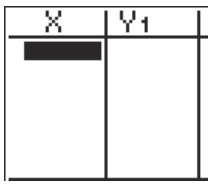
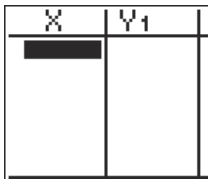
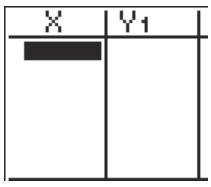
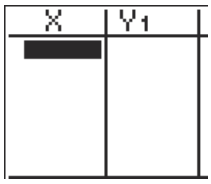
Step	Key strokes	Your screen should look like…
1. Set up your table using TBLSET	2nd WINDOW	
2. Input your equation	Y= $Y1 = (1/3)\ \pi\ (x)^2\ (12)$ The (1/3) represents one-third	
3. View the table.	2nd GRAPH Press DEL until the table is clear.	
4. Use guess and check until the volume (Y1) is 316 cm³.	You will have to keep guessing and checking until you get your answer of Y1 = 316 cm³ which is a volume of 316 cm³.	

Repeat this process for each equation on the next page.

Solving Measurement Equations With The Graphing Calculator.

Complete the table below by connecting the algebraic representation to the appropriate measurement formula.

					
$V = lwh$	$V = \frac{1}{3}lwh$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{1}{2}lwh$	$V = \frac{4}{3}\pi r^3$

Equation	Draw shape and label the dimensions.	Name the unknown dimension.	Answer showing table entries from TI-73
a. $100 = \pi (7)^2 (x)$			
b. $250 = (7)(12)(x)$			
c. $50 = \frac{4}{3}\pi x^3$			
d. $354 = \frac{1}{3}\pi (15)^2 (x)$			
e. $2500 = \pi (x)^2 (30)$			
f. $375 = \frac{1}{2}\left(\frac{3}{3}\pi (x)^3\right)$			

Station 3: *Things* and Gold

Name: _____

How much is that Worth!

Your Task

Select 5 geo-solids from the envelope and determine how many loaves of bread you could purchase with the 5 geo-solids if they were made out of solid (24 K) gold.

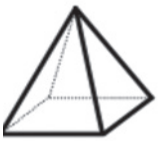
Information You Need

Gold is quite pricey at about $40.00 per gram. It is also incredibly heavy with a density of 19 grams per cm^3. To compare, the plastic 1 cm^3 blocks at this learning station each have a mass of about 1.5 grams (their density is 1.5 grams per cm^3). Bread costs about $2.00 per loaf.

Before you determine the volume of each geo-solid, please use the 1 cm_ blocks to estimate the volume. Why is it important to estimate the volume first before doing calculations?

Show all your work on the paper provided. Include labeled diagrams, your estimates, formulas and calculations. Keep your work organized. Clear communication is important.

Good luck.

					
$V = lwh$	$V = \frac{1}{3}lwh$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{1}{2}lwh$	$V = \frac{4}{3}\pi r^3$